



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 The matrix $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{M}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





DO NOT WRITE IN THIS MARGIN

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The unit square S in the x - y plane is transformed by $\mathbf{M}$ onto the rectangle P .

- (c) Find the matrix which transforms P onto S . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



[illegible]

* 0000800000005 *



5

BLANK PAGE





3 The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....




$$\sum_{r=1}^{10} ((\alpha^2 + r)^2 + (\beta^2 + r)^2 + (\gamma^2 + r)^2 + (\delta^2 + r)^2). \quad [5]$$
[illegible]



4 Let $w_r = r(r+1)(r+2)\dots(r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2)\dots(r+9). \quad [2]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that $u_r = (r+1)(r+2)\dots(r+9)$, find $\sum_{r=1}^n u_r$ in terms of n . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....




$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

[illegible]



5 The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





- 6 The curve C has equation $y = \frac{x^2 + a}{x + a}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find, in terms of a , the x -coordinates of the stationary points on C .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



(c) Sketch C , stating the coordinates of any intersections with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{x^2 + a}{x + a} \right|$.

[1]



DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

.....

.....

.....

.....

.....

25

9231/13/M/J/25





7 The curve C has polar equation $r^2 = e^{\sin \theta} \cos \theta$, for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$.

- (a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





(c) Sketch C .

[3]

(d) Find the area of the region bounded by C , giving your answer in exact form.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no text or other markings.





Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

